

USMT-02 - Mathematics Paper-II - Differential Calculus and Trigonometry

P. Pages : 2

Time : Three Hours

**GUG/S/25/11557**

Max. Marks : 60

- Notes : 1. Solve all the **five** questions.
2. Each questions carry equal marks.

UNIT - I

1. a) Prove that if limit of a function $f(x, y)$ as $(x, y) \rightarrow (x_0, y_0)$ exists, then it is unique. **6**
- b) Let $f(x, y) = \frac{x^3}{x^2 + y^2}$, for $(x, y) \neq (0, 0)$ **6**
 $= 0$, for $(x, y) = (0, 0)$
 Prove that f is continuous at $(0, 0)$.

OR

- c) If $u = 3(ax + by + cz)^2 - (x^2 + y^2 + z^2)$ and $a^2 + b^2 + c^2 = 1$ then show that **6**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$
- d) If $u = F(x - y, y - z, z - x)$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$. **6**

UNIT - II

2. a) If $u = f(x, y)$ be a homogeneous function of degree n in x, y then prove that **6**

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu$$
- b) If $x = r \sin \theta \cos \phi, y = r \sin \theta \sin \phi, z = r \cos \theta$ then find $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$. **6**

OR

- c) Obtain the expansion of $f(x, y) = x^2 - y^2 + 3xy$ at the point $(1, 2)$. **6**
- d) Find the stationary points of $x^4 + y^4 - 2x^2 + 4xy - 2y^2$. **6**

UNIT - III

3. a) Find the equations of the tangent and the normal to the curve $y = x^2$ from the point $(-1, 3)$. **6**

- b) Prove that radius of curvature of the curve $y = f(x)$ at any point $p, (x, y)$ is given by 6

$$\rho = \frac{(1 + y'^2)^{3/2}}{y''}$$

OR

- c) Find the asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 = 1$. 6

- d) Trace the curves- 6

i) $y^2(a - x) = x^3$ ii) $x^2y^2 = a^2(y^2 - x^2)$.

UNIT - IV

4. a) If $x_r = \cos\left(\frac{\pi}{2^r}\right) + i \sin\left(\frac{\pi}{2^r}\right)$ then prove that $x_1 x_2 x_3 \dots \infty = -1$. 6

- b) Prove that $(1 + i)^n + 1(1 - i)^n = 2^{\left(\frac{n}{2}\right)+1} \cos\left(\frac{n\pi}{4}\right)$ 6

OR

- c) Find all the values of $(1 + i)^{1/4}$ 6

- d) If $\sin(\alpha + i\beta) = x + iy$ then $\frac{x^z}{\cosh^2 \beta} + \frac{y^2}{\sinh^2 \beta} = 1$ and $\frac{x^2}{\sin^2 \alpha} - \frac{y^2}{\cos^2 \alpha} = 1$ 6

5. Solve **any six** questions.

- a) Write $\epsilon - \delta$ definition of limit of a function of two variables. 2
- b) If $z = f(xy)$, show that $xz_x - yz_y = 0$. 2
- c) If $x = r \cos \theta, y = r \sin \theta$, find $\frac{\partial(x, y)}{\partial(r, \theta)}$. 2
- d) Define stationary point of $f(x, y)$. 2
- e) Find the tangent and normal at $(1, 3)$ to the curve $y = x^3 + 2$. 2
- f) Find the radius of curvature at any point on the curve $s = c \tan \psi$. 2
- g) Express $1 + i$ in polar form. 2
- h) Prove that $\sin iz = i \sinh z$. 2
